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## *Postscript to Note on a Point in Vulgar Fractions.*

BY J. J. SYLVESTER.

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LET  $\phi x$  represent  $x^2 - x + 1$ ,  $\phi^n c$  will then be the general term of the “limiting sorites” whose first term is  $c$ , for which, if we please,  $1 - c$  may be substituted. The properties of the numbers  $\phi^n c$  seem to be worthy of some attention. I confine my observations in what follows to the lowest of such series, viz. where  $c = 2$  or  $-1$ .

The first five terms in such series then become  $\bar{1}$  or  $2, 3, 7, 43, 1807, 3263443$ , of which all but  $1807$ , which  $= 13.139$ , are prime numbers. Every term in the series must contain only factors of the form  $6i + 1$ , and this, joined to the fact that a prime factor which has once appeared in any term can never reappear in any other, favors a tendency, so to say, of the numbers to remain primes, or at all events, to be of very limited frangibility into a product of primes.

It is easy to determine whether any proposed prime can occur as a factor of any term whatever in the series; for taking that number, say  $p$ , as a modulus, if  $r$  is a remainder of any term to that modulus, the remainder of the next term will be  $r^2 - r + 1$ , and as soon as any remainder reappears the series of remainders becomes periodic; so that necessarily in less than the number  $p$  of remainders, if  $p$  does divide any term of the sorites, we must arrive at a remainder zero, subsequent to which all the remainders are unity. I give the remainders and periods in the annexed table for all values of  $p$  of the form  $6i + 1$  up to  $139$ , from which it will be seen that, under that limit,  $13$  and  $73$  are the only prime numbers which are contained as factors in the terms of the series.

$p$	Remainders of $\phi^n(2)$ to modulus $p$ .
2	0.
3	2, 0.
7	2, 3, 0.
13	2, 3, 7, 4, 0.
19	2, 3, 7, 5; 2, 3, 7, 5; ....
31	2, 3, 7, 12, 9, 11, 18, 28, 13; 2, 3, 7, ...., 13; ....
37	2, 3, 7; 6, 31; 6, 31; ....
43	2, 3, 7, 0.
61	2, 3, 7, 43, 38, 4, 13, 35, 32; 17, 29, 20, 15, 28, 25, 52, 30; ....
67	2, 3; 7, 43, 65; 7, 43, 65; ....
73	2, 3, 7, 43, 55, 51, 69, 21, 56, 15, 65, 0.
79	2, 3, 7; 43, 69, 32, 45, 6, 31, 61, 27, 71, 73; ....
97	2, 3, 7, 43, 61; 72, 69, 37; 72, 69, 37; ....
103	2, 3; 7, 43, 56, 94, 91, 54, 82, 51, 79, 86, 101; 7, 43, ....; ....
109	{ 2, 3, 7, 43, 63, 92, 89, 94, 23, 71, 66, 40, 35, 101, 73, 25, 56, 29, 50, 53; 32, 12, 24, 8; 32, 12, 24, 8; .... }
127	2, 3, 7, 43, 29, 51, 11; 111, 19, 89, 86, 72, 33, 41, 117; ....
139	2, 3, 7, 43, 0.
151	2, 3, 7, 43, 146; 31, 25, 148, 13, 6; ....
157	2, 3; 7, 43, 80, 41, 71, 104, 37, 77, 44, 9, 73, 76, 49, 155; ....
163	2, 3; 7, 43, 14, 20, 55, 37, 29, 161; ....
181	2, 3, 7, 43, 178, 13, 157, 58, 49, 0.
193	2; 3, 7, 43, 70, 6, 31, 159, 33, 92, 74, 192; ....
199	2, 3; 7, 43, 16, 42, 131, 116, 8, 57, 9, 73, 83, 41, 49, 164, 67, 45, 190, 91, 32, 197; ....